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TECHNICAL NOTES

Flow and heat transfer of plane surfaces moving in parallel and reversely to the free stream

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1. INTRODUCTION

TWO TYPES of classical forced convection problems have been extensively studied for the past years. They are the Blasius problem [1, 2] of a steady flow over a stationary flat plate; and the Sakiadis problem [3, 4] of a plate moving continuously in a quiescent ambient fluid. However, in many practical engineering systems both the plane surface and the ambient fluid are moving in parallel. Examples are the cooling of polymer films or sheets and metallic plates on conveyers. The problem of a moving surface in a parallel free stream has been investigated by Abdelhafez [5]. He obtained numerical solutions of the governing boundary layer equations and Navier–Stokes equations. Chappidi and Gunnerson [6] studied the same problem by using an integral method along with a perturbation procedure.

In this paper, we study the general forced convection problem of a surface moving continuously in a flowing stream by using a quite different transformation to obtain similarity solutions. The present solutions are very accurate for any relative speed of the surface and the free stream over a wide range of Prandtl number between 0.01 and 10 000. We study not only the case of a plane surface moving in parallel to the free stream but also the case of a surface moving reversely. The latter case has not been studied previously. For application convenience, very accurate correlation equations are proposed for predicting the wall friction and heat transfer for any velocity ratio of the surface and the free stream.

2. FLOW PROBLEM

2.1. *Flow equations*

Consider a plane surface moves, in parallel or reversely to a free stream of uniform velocity u_∞, at a constant velocity u_w. Either the surface velocity or the free stream velocity may be zero but not both. Assume the surface and the free stream are at the same temperature or with small temperature difference so that the buoyancy effect on flow is negligible. The physical properties of fluid are assumed to be constant. Under such conditions, the continuity and momentum equations of the steady, laminar boundary-layer flow on the moving surface are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}. \tag{2}$$

The boundary conditions of this system are

$$u=\pm u_w,\ v=0 \text{ at } y=0, \tag{3}$$

and

$$u=u_\infty \text{ as } y\to 0. \tag{4}$$

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The boundary condition of $u=+u_w$ in equation (3) represents the case of a plane surface moving in parallel to the free stream, while $u=-u_w$ represents the case of a surface moving reversely.

In order to analyse the effect of both the moving surface and the free stream on the boundary-layer flow, we propose a new similarity coordinate and a dimensionless stream function as

$$\eta=(y/x)(Re_W+Re_\infty)^{1/2},\quad f=\psi/\nu(Re_W+Re_\infty)^{1/2} \tag{5}$$

which are the combinations of the traditional ones:

$$\eta_B=(y/x)Re_\infty^{1/2},\quad f_B=\psi/\nu Re_\infty^{1/2} \tag{6}$$

for the Blasius problem; and

$$\eta_S=(y/x)R_W^{1/2},\quad f_S=\psi/\nu Re_W^{1/2} \tag{7}$$

for the Sakiadis problem, where the Reynolds numbers

$$Re_W=u_W x/\nu,\quad Re_\infty=u_\infty x/\nu. \tag{8}$$

In addition, a parameter of velocity ratio is introduced as

$$\gamma=u_W/(u_w+u_\infty)=(1+u_\infty/u_W)^{-1}=(1+Re_\infty/Re_W)^{-1}. \tag{9}$$

Note that for the Blasius problem, $u_W=0$, therefore $\gamma=0$. On the other hand, for the Sakiadis problem, $u_\infty=0$, and thus $\gamma=1$.

The present and the traditional transformation variables defined in equations (5)–(7) resulted in the same similarity equation

$$2f_{\eta\eta\eta}+ff_{\eta\eta}=0. \tag{10}$$

However, the transformed boundary conditions of the present system are

$$f(0)=0,\quad f_\eta(0)=\pm\gamma,\quad f_\eta(\infty)=1-\gamma \tag{11}$$

which can be readily reduced to the conventional boundary conditions for the Blasius problem and the Sakiadis problem by setting $\gamma=0$ and 1, respectively.

Equations (10) and (11) were integrated numerically by using the fourth order Runge–Kutta scheme along with the shooting method. Since equation (10) is a similarity equation, very accurate numerical solutions can be obtained. The high accuracy of the numerical results has been verified by comparing the present data of $f_{\eta\eta}(0)$ for $\gamma=0$ and 1 with the reported solutions of th Blasius problem and the Sakiadis problem.

2.2. *Velocity profiles*

The dimensionless stream-wise velocity

$$u/(u_W+u_\infty)=f_\eta(\eta) \tag{12}$$

is presented in Fig. 1(a) for the parallel moving surface; and in Fig. 1(b) for the reverse moving surface. Figure 1(a) shows clearly a gradual conversion of the velocity profiles as the parameter of velocity ratio γ increases from 0 to 1. In Fig.

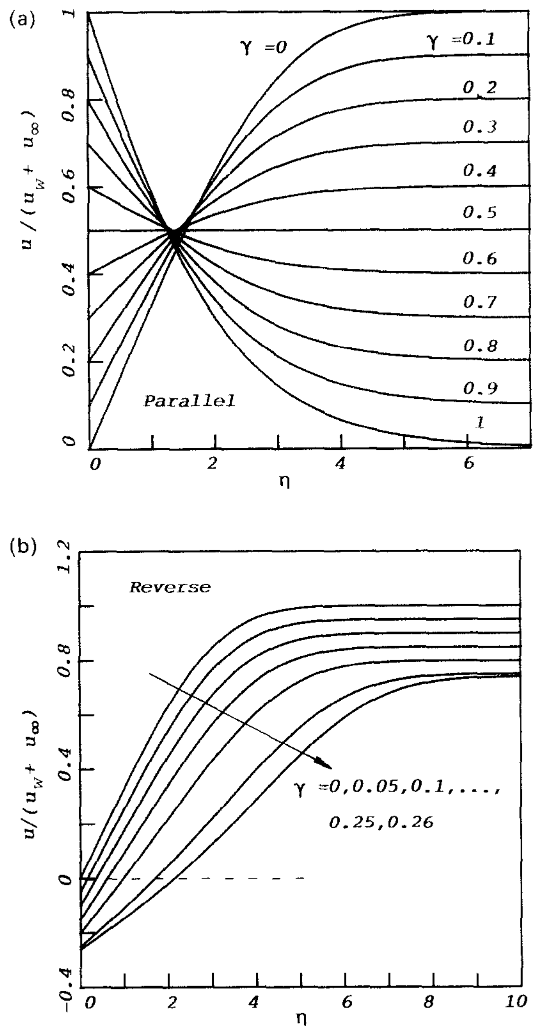

FIG. 1. Profiles of the dimensionless velocity $u/(u_W+u_\infty)$, (a) parallel moving surface; (b) reverse moving surface.

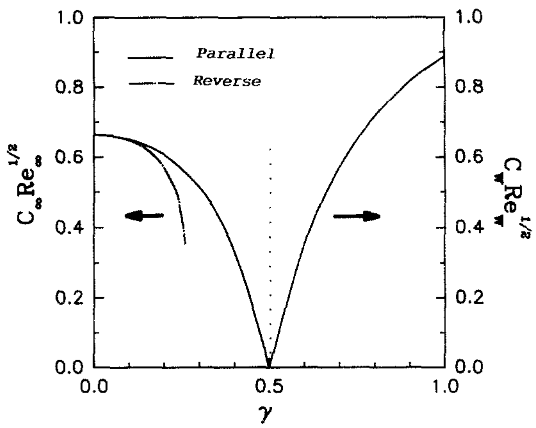

FIG. 2. Variations of wall friction with γ.

1(b), a reverse flow region near the plane surface (at the region of small η) can be seen.

2.3. *Wall shear stress*

The wall shear stress is related to $f_{\eta\eta}(0)$ by the relation

$$\tau_W = \rho\nu\left|\left(\frac{\partial u}{\partial y}\right)_{y=0}\right| = \rho(\nu/x)^2(Re_W + Re_\infty)^{1/2}|f_{\eta\eta}(0)|. \tag{13}$$

To express the wall shear stress nondimensionally, we define two types of the local friction coefficients as

$$C_\infty = 2\tau_W/\rho u_\infty^2, \quad C_W = 2\tau_W/\rho u_W^2. \tag{14}$$

A combination of equations (13) and (14) gives

$$C_\infty Re_\infty^{1/2} = 2(1-\gamma)^{-3/2}|f_{\eta\eta}(0)|, \quad C_W Re_W^{1/2} = 2\gamma^{-3/2}|f_{\eta\eta}(0)|. \tag{15}$$

Figure 2 shows that, for the parallel moving surface, $C_\infty Re_\infty^{1/2}$ decreases from 0.66412 to 0 as γ increases from 0 to 0.5; and $C_W Re_W^{1/2}$ increases from 0 to 0.88753 as γ increases from 0.5 to 1. For the reverse moving surface, $C_\infty Re_\infty^{1/2}$ decreases as γ increases from 0 to a limit of about 0.26. Beyond this limit, the laminar boundary layer would break-down due to the opposite moving of the surface and the free stream, and thus the convergent numerical solutions cannot be obtained. This figure also shows that the wall friction of the reverse moving surface is smaller than that of the parallel moving surface at the same speed.

2.4. *Correlations of the friction coefficient*

For application convenience, we propose the following correlation equations for predicting the friction coefficient of the parallel moving surface:

$$C_\infty Re_\infty^{1/2} = 0.66412(1-2\gamma)/(1-1.14\gamma)^{3/2}, \quad 0 \leqslant \gamma \leqslant 0.5 \tag{16}$$

and

$$C_W Re_W^{1/2} = 0.88753(2\gamma-1)/(1.036\gamma)^{3/2}, \quad 0.5 \leqslant \gamma \leqslant 1. \tag{17}$$

The maximum errors of the correlation equations (16) and (17) are less than 3.4 and 5.2%, respectively, when compared with the numerical data.

The correlation equation of the reverse moving surface is developed, based on $C_\infty Re_\infty^{1/2} = 0.66412$ at $\gamma = 0$ and 0.43312 at $\gamma = 0.25$, as

$$C_\infty Re_\infty^{1/2} = (1-\gamma)^{-3/2}[(0.66412)^n(1-4\gamma)^n + (1.1253\gamma)^n]^{1/n}, \quad 0 \leqslant \gamma \leqslant 0.25. \tag{18}$$

The exponent n in equation (18) can be determined by comparing with the numerical data. The maximum error of this correlation equation with $n = 0.86$ does not exceed 2.2%.

3. HEAT TRANSFER PROBLEM

3.1. *Energy equation and transformation*

The boundary-layer energy equation and boundary conditions of the considered system are the same as the conventional ones. If we applied the dimensionless variables defined in equation (5) to the transformation of the energy equation, the transformed equation can only be solved for fluids of Prandtl number between 0.1 and 10. In order to obtain the convergent solutions for a wide range of Prandtl number ($0.01 \leqslant Pr \leqslant 10\,000$), we introduce

$$\lambda = (\sigma Re_W + \omega Re_\infty)^{1/2}, \quad \text{where}$$

$$\sigma = \frac{Pr^2}{1+Pr}, \quad \omega = \frac{Pr}{(1+Pr)^{1/3}} \tag{19}$$

to define the dimensionless coordinate and dimensionless stream function as

$$\zeta = (y/x)\lambda, \quad F = \psi/\alpha\lambda. \tag{20}$$

In addition, we modify the parameter of relative velocity γ to a new one:

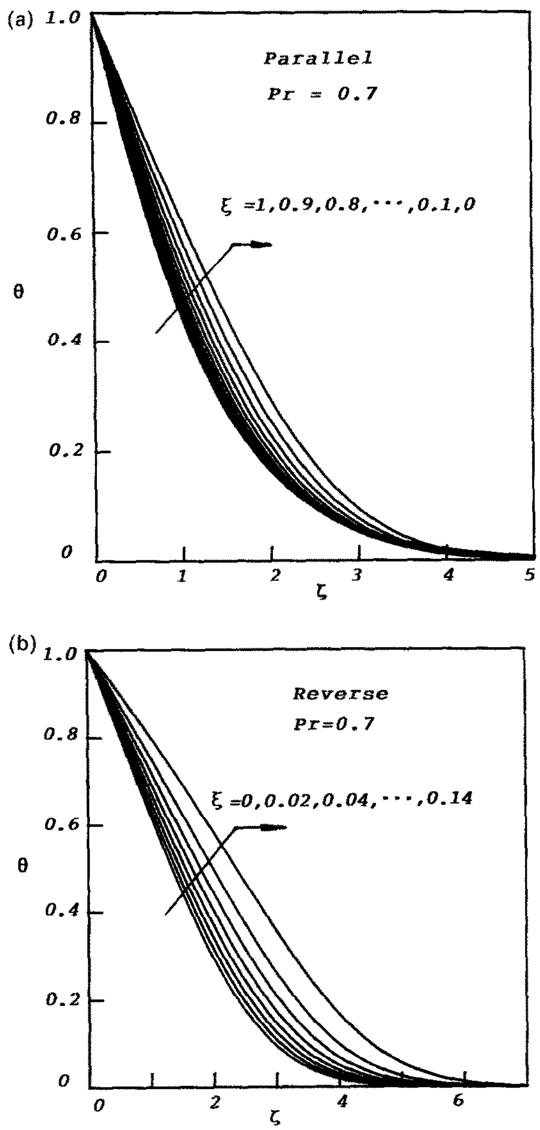

FIG. 3. Dimensionless temperature profiles, (a) parallel moving surface; (b) reverse moving surface.

$$\xi = (1+\omega u_x/\sigma u_W)^{-1} = (1+\omega Re_x/\sigma Re_W)^{-1}. \quad (21)$$

Using the proper dimensionless variables defined in equations (19)–(21), the transformed momentum equation and boundary conditions becomes

$$2PrF''' + FF'' = 0, \quad (22)$$

$$F(0) = 0, \quad F'(0) = \pm\xi(1+Pr)/Pr,$$

$$F'(\infty) = (1-\xi)(1+Pr)^{1/3} \quad (23)$$

where the primes denote differentiation with respect to ζ. The transformed energy equation and boundary conditions are

$$2\theta'' + F\theta' = 0, \quad (24)$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \quad (25)$$

where the dimensionless temperature $\theta = (T-T_\infty)/(T_W-T_\infty)$.

The similarity momentum equation (22) was solved accurately by using Runge–Kutta integration scheme along with the shooting method. The transformed energy equation (24) was also solved by the same numerical procedure. The similarity solutions has been verified to be very accurate when compared with the reported data of the Blasius problem and the Sakiadis problem.

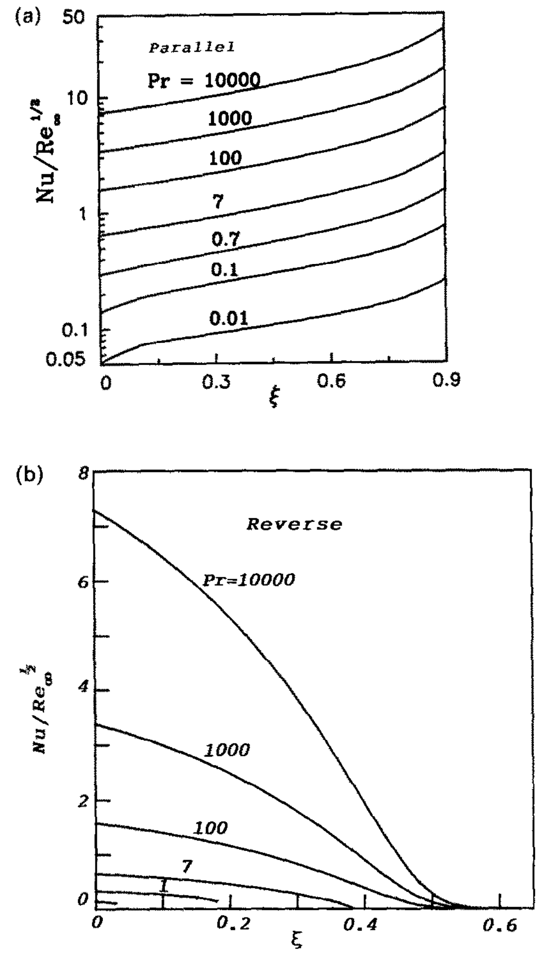

FIG. 4. Variations of $Nu/Re_x^{1/2}$ with ξ, (a) parallel moving surface; (b) reverse moving surface.

3.2 *Temperature profiles*

Representative dimensionless temperature profiles of gases ($Pr = 0.7$) on a parallel moving surface are presented in Fig. 3(a). This figure shows that the fluid temperature decreases with increasing surface velocity.

The dimensionless temperature profiles for the case of a reverse moving surface are shown in Fig. 3(b) for some specified values of ξ. As can be seen from this figure, the fluid temperature increases with increasing surface velocity.

3.3. *Local Nusselt number*

The local Nusselt number, $Nu = hx/k$, can be obtained from the numerical results by the relation

$$\frac{Nu}{Re_x^{1/2}} = -\left(\frac{\omega}{1-\xi}\right)^{1/2}\theta'(0). \quad (26)$$

The variations of $Nu/Re_x^{1/2}$ with the relative velocity parameter ξ for $Pr = 0.01$ to 10 000 are presented in Fig. 4(a). This figure shows that $Nu/Re_x^{1/2}$ increases with increasing ξ. In addition, the values of $Nu/Re_x^{1/2}$ for large Pr are greater than those for small Pr as can be seen from this figure.

Table 1. Values of n and the maximum error of the heat transfer correlation equation over the whole range of velocity ratio

Range of Pr	Values of n	Maximum error (%)
$0.01 \leqslant Pr \leqslant 0.1$	0.76	5.7
$0.1 \leqslant Pr \leqslant 0.7$	0.84	6.5
$0.7 \leqslant Pr \leqslant 7$	0.97	5.2
$7 \leqslant Pr \leqslant 10\,000$	1.02	1.4

For the case of a reverse moving surface, Fig. 4(b) shows that $Nu/Re_x^{1/2}$ decreases as ξ increases. The decrease of the Nusselt number is due to the back flow of hot fluid from the down-stream.

3.4. *Correlation equations of heat transfer*

A correlation equation of the local Nusselt number for any velocity ratio is developed as

$$\left(\frac{Nu}{\lambda}\right)^n = \left[(1-\xi)\frac{Nu_B}{(\omega Re_x)^{1/2}}\right]^n + \left[\xi\frac{Nu_S}{(\sigma Re_W)^{1/2}}\right]^n. \quad (27)$$

This correlation can be rewritten as

$$\frac{Nu}{(\omega Re_x)^{1/2}} = (1-\xi)^{1/2}\left[\left(\frac{Nu_B}{(\omega Re_x)^{1/2}}\right)^n + \left(\frac{\xi}{1-\xi}\frac{Nu_S}{(\sigma Re_W)^{1/2}}\right)^n\right]^{1/n} \quad (28)$$

where $Nu_B/(\omega Re_x)^{1/2}$ for the special case of the Blasius problem ($\gamma = 0$) can be estimated from the correlation in ref. [7]:

$$Nu_B/Re_x^{1/2} = 0.3386Pr^{1/2}/(0.0526+0.1121\,Pr^{1/2}+Pr)^{1/6}. \quad (29)$$

The maximum error of this correlation does not exceed 1.4% for $0.001 \leqslant Pr \leqslant \infty$. While $Nu_S/(\sigma Re_W)^{1/2}$ for the special case of the Sakiadis problem ($\gamma = 1$) can be predicted by the present correlation equation

$$Nu_S/Re_W^{1/2} = 0.5642Pr^{1/2}/(0.4621+0.1395Pr^{1/2}+Pr)^{1/2}. \quad (30)$$

The maximum error of this correlation is less than 1% for $0.01 \leqslant Pr \leqslant 10\,000$.

Appropriate values of the exponent n in the correlation equations (27) and (28) for different Pr are presented in Table 1. Predictions of heat transfer from the correlation coincide satisfactorily with the numerical results, as indicated in this table.

4. CONCLUSIONS

This paper studied the general convection problem of a continuous moving surface in a flowing fluid by introducing novel transformation variables and parameters of velocity ratio. For the case of a plane surface moving in parallel to a free stream, very accurate similarity solutions and correlation equations for predicting the wall friction and heat transfer rate have been obtained for any ratio of surface velocity and free stream velocity over the range of $0.01 \leqslant Pr \leqslant 10\,000$. The case of a surface moving in the reverse direction of the free stream has also been analyzed. Velocity and temperature profiles have been presented to show the effects of the relative motion of the plane surface and the free stream. The developed analysis method can be applied to the mixed convection problems and many others.

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Thermal entrance length and Nusselt numbers in coiled tubes

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INTRODUCTION

SECONDARY flow in coiled tubes, generated as a result of tube curvature, significantly increases heat transfer as compared to flow in straight tubes. In straight tubes the peripherally averaged Nusselt number, $\langle Nu\rangle_\phi$, is a maximum at the tube inlet, decreases monotonically in the downstream direction, and asymptotes to a fully developed value. In coiled tubes on the other hand, $\langle Nu\rangle_\phi$ undergoes spatial oscillations before settling down to a fully developed value. Numerical calculations of this phenomenon have been carried out by Dravid *et al.* [1], Tarbell and Samuels [2], Patankar *et al.* [3], Akiyama and Cheng [4] and Janssen and Hoogendoorn [5]. There is also some experimental evidence in the observed spatial wall temperature oscillations reported by Dravid *et*